|  | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: |
| 1 | $\frac{2 x}{x+1}-\frac{1}{x-1}=1$ |  |  |
|  | $\Rightarrow 2 x(x-1)-(x+1)=(x+1)(x-1)$ | M1 | mult throughout by $(x+1)(x-1)$ or combining fractions and mult up oe (can retain denominator throughout). Condone a single computational error provided that there is no conceptual error. Do not condone omission of brackets unless it is clear from subsequent work that they were assumed. |
|  | $\Rightarrow 2 x^{2}-3 x-1=x^{2}-1$ | A1 | any fully correct multiplied out form (including say, if 1's correctly cancelled) soi |
|  | $\Rightarrow \quad x^{2}-3 x=0=x(x-3)$ | DM1 | dependent on first M1.For any method leading to both solutions. Collecting like terms and forming quadratic $(=0)$ and attempting to solve *(provided that it is a quadratic and $b^{2}-4 a c \geq 0$ ). Using either correct quadratic equation formula (can be error when substituting), factorising (giving correct $x^{2}$ and constant terms when factors multiplied out), completing the square oe soi.* |
|  | $\Rightarrow \quad x=0$ or 3 | A1 | for both solutions www. |
|  |  |  | SC B1 for $x=0$ (or $x=3$ ) without any working <br> SC B2 for $x=0$ (or $x=3$ ) without above algebra but showing that they satisfy equation <br> SC M1A1M0 SCB1 for first two stages followed by stating $x=0$ <br> SC M1A1M0 SCB1 for first two stages and cancelling $x$ to obtain $x=3$ only. |
|  |  | [4] |  |



| 3 | $\frac{3 x+2}{x\left(x^{2}+1\right)}=\frac{A}{x}+\frac{B x+C}{\left(x^{2}+1\right)}$ |  |  |
| :--- | :--- | :--- | :--- |
| $\Rightarrow \quad 3 x+2=A\left(x^{2}+1\right)+(B x+C) x$ | M1 | correct partial fractions |  |
| $x=0 \Rightarrow 2=A$ <br> coefft of $x^{2}: 0=A+B \Rightarrow B=-2$ <br> coefft of $x: 3=C$ | M1 |  |  |
| $\Rightarrow \quad \frac{3 x+2}{x\left(x^{2}+1\right)}=\frac{2}{x}+\frac{3-2 x}{\left(x^{2}+1\right)}$ | M1 | equating coefficients |  |
| $\Rightarrow$ | A1 least one of $B, C$ correct |  |  |
|  |  |  |  |


| $4 \begin{aligned} \frac{4}{x\left(x^{2}+4\right)} & =\frac{A}{x}+\frac{B x+C}{x^{2}+4} \\ & =\frac{A\left(x^{2}+4\right)+(B x+C) x}{x\left(x^{2}+4\right)} \end{aligned}$ | M1 | correct partial fractions |
| :---: | :---: | :---: |
| $\Rightarrow \quad 4=A\left(x^{2}+4\right)+(B x+C) x$ | M1 |  |
| $x=0 \Rightarrow 4=4 A \Rightarrow A=1$ | B1 | $A=1$ |
| coefft of $x^{2}$ : $0=A+B \Rightarrow B=-1$ | DM1 | Substitution or equating coeffts |
| coeffts of $x$ : $0=C$ | A1 | $B=-1$ |
| $\Rightarrow \quad 4 \quad-1-x$ | A1 | $C=0$ |
| $\Rightarrow \quad \overline{x\left(x^{2}+4\right)}=\frac{-}{x}-\overline{x^{2}+4}$ | [6] |  |

$$
\begin{array}{ll}
5 & \frac{2 x}{x-2}-\frac{4 x}{x+1}=3 \\
\Rightarrow & 2 x(x+1)-4 x(x-2)=3(x-2)(x+1) \\
\Rightarrow & 2 x^{2}+2 x-4 x^{2}+8 x=3 x^{2}-3 x-6 \\
\Rightarrow & 0=5 x^{2}-13 x-6 \\
& =(5 x+2)(x-3) \\
\Rightarrow & x=-2 / 5 \text { or } 3 .
\end{array}
$$

M1
M1 A1

M1

## Clearing fractions expanding brackets <br> oe <br> factorising or formula oe

| Question |  | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: | :--- |
| $\mathbf{6}$ | (i) | $\frac{x}{(1+x)(1-2 x)}=\frac{A}{1+x}+\frac{B}{1-2 x}$ <br> $\Rightarrow \quad x=A(1-2 x)+B(1+x)$ <br> $x=1 / 2 \Rightarrow 1 / 2=B(1+1 / 2) \Rightarrow B=1 / 3$ <br> $x=-1 \Rightarrow-1=3 A \Rightarrow A=-1 / 3$ | M1 | A1 <br> expressing in partial fractions of correct form (at any stage) and <br> attempting to use cover up, substitution or equating coefficients <br> Condone a single sign error for M1 only. <br> www cao |
| A1 | www cao <br> (accept $\mathrm{A} /(1+\mathrm{x})+\mathrm{B} /(1-2 \mathrm{x}), \mathrm{A}=-1 / 3, \mathrm{~B}=1 / 3$ as sufficient for <br> full marks without needing to reassemble fractions with numerical <br> numerators) |  |  |  |


| Question |  | Answer | Marks | Guidance |
| :---: | :---: | :---: | :---: | :---: |
| 6 | (ii) | $\begin{aligned} & \frac{x}{(1+x)(1-2 x)}=\frac{-1 / 3}{1+x}+\frac{1 / 3}{1-2 x} \\ & =\frac{1}{3}\left[(1-2 x)^{-1}-(1+x)^{-1}\right] \\ & =\frac{1}{3}\left[1+(-1)(-2 x)+\frac{(-1)(-2)}{2}(-2 x)^{2}+\ldots-\left(1+(-1) x+\frac{(-1)(-2)}{2} x^{2}+\ldots\right)\right] \\ & =\frac{1}{3}\left[1+2 x+4 x^{2}+\ldots-\left(1-x+x^{2}+\ldots\right)\right] \end{aligned}$ $=\frac{1}{3}\left(3 x+3 x^{2}+\ldots\right)=x+x^{2}+\ldots \text { so } a=1 \text { and } b=1$ | M1 <br> A1 <br> A1 <br> A1 | correct binomial coefficients throughout for first three terms of either $(1-2 x)^{-1}$ or $(1+x)^{-1}$ oe ie $1,(-1),(-1)(-2) / 2$, not nCr form. Or correct simplified coefficients seen. $1+2 x+4 x^{2}$ <br> $1-x+x^{2} \quad$ (or $1 / 3 /-1 / 3$ of each expression, ft their $A / B$ ) <br> If $k\left(1-x+x^{2}\right)$ (A1) not clearly stated separately, condone absence of inner brackets (ie $1+2 x+4 x^{2}-1-x+x^{2}$ ) only if subsequently it is clear that brackets were assumed, otherwise A1A0. <br> [ie $-1-x+x^{2}$ is A 0 unless it is followed by the correct answer] Ignore any subsequent incorrect terms or from expansion of $x(1-2 x)^{-1}(1+x)^{-1}$ www cao |
|  |  | OR $\begin{aligned} x(1-x-2 & \left.x^{2}\right)=x\left(1-\left(x+2 x^{2}\right)\right) \\ & =x\left(1+x+2 x^{2}+(-1)(-2)\left(x+2 x^{2}\right)^{2} / 2+\ldots \ldots \ldots\right) \\ & =x\left(1+x+2 x^{2}+x^{2} \ldots \ldots \ldots\right) \\ & =x+x^{2} \ldots . . \text { so } a=1 \text { and } b=1 \end{aligned}$ | M1 <br> A2 <br> A1 | correct binomial coefficients throughout for (1-( $\left.\mathrm{x}+2 \mathrm{x}^{2}\right)$ ) oe (ie $1,-1$ ), at least as far as necessary terms ( $1+\mathrm{x}$ ) (NB third term of expansion unnecessary and can be ignored) $x(1+x) \text { www }$ <br> ww cao |
|  |  | Valid for $-1 / 2<x<1 / 2$ or $\|x\|<1 / 2$ | B1 <br> [5] | independent of expansion. Must combine as one overall range. condone $\leq \mathrm{s}$ (although incorrect) or a combination. Condone also, say $-1 / 2<\|x\|<1 / 2$ but not $x<1 / 2$ or $-1<2 x<1$ or $-1 / 2>x>1 / 2$ |


| $\begin{aligned} 7 \text { (i) } \quad & v=\int 10 e^{-\frac{1}{2} t} d t \\ & =-20 e^{-\frac{1}{2} t}+c \\ & \text { when } t=0, v=0 \\ \Rightarrow & 0=-20+c \\ \Rightarrow \quad & c=20 \\ & \text { so } v=20-20 e^{-\frac{1}{2} t} \end{aligned}$ | M1 <br> A1 <br> M1 <br> A1 <br> [4] | separate variables and intend to integrate $\begin{aligned} & -20 e^{-\frac{1}{2} t} \\ & \text { finding } c \end{aligned}$ <br> cao |
| :---: | :---: | :---: |
| (ii) As $t \rightarrow \infty \quad \mathrm{e}^{-1 / 2 t} \rightarrow 0$ $\Rightarrow \quad v \rightarrow 20$ <br> So long term speed is $20 \mathrm{~m} \mathrm{~s}^{-1}$ | M1 <br> A1 [2] | ft (for their $c>0$, found) |
| $\begin{aligned} & \text { (iii) } \begin{aligned} & \frac{1}{(w-4)(w+5)}=\frac{A}{w-4}+\frac{B}{w+5} \\ &=\frac{A(w+5)+B(w-4)}{(w-4)(w+5)} \\ & \Rightarrow \quad 1 \equiv A(w+5)+B(w-4) \end{aligned} \\ & \begin{aligned} & w=4: 1=9 A \Rightarrow A=1 / 9 \\ & w=-5: 1=-9 B \Rightarrow B=-1 / 9 \\ & \Rightarrow \frac{1}{(w-4)(w+5)}=\frac{1 / 9}{w-4}-\frac{1 / 9}{w+5} \\ &=\frac{1}{9(w-4)}-\frac{1}{9(w+5)} \end{aligned} \end{aligned}$ | M1 <br> M1 <br> A1 <br> A1 <br> [4] | cover up, substitution or equating coeffs $\begin{aligned} & 1 / 9 \\ & -1 / 9 \end{aligned}$ |
| $\begin{aligned} & \text { (iv) } \frac{d w}{d t}=-\frac{1}{2}(w-4)(w+5) \\ & \Rightarrow \quad \int \frac{d w}{(w-4)(w+5)}=\int-\frac{1}{2} d t \\ & \Rightarrow \int\left[\frac{1}{9(w-4)}-\frac{1}{9(w+5)}\right] d w=\int-\frac{1}{2} d t \\ & \Rightarrow \frac{1}{9} \ln (w-4)-\frac{1}{9} \ln (w+5)=-\frac{1}{2} t+c \\ & \Rightarrow \frac{1}{9} \ln \frac{w-4}{w+5}=-\frac{1}{2} t+c \\ & \text { When } t=0, w=10 \Rightarrow c=\frac{1}{9} \ln \frac{6}{15}=\frac{1}{9} \ln \frac{2}{5} \\ & \Rightarrow \ln \frac{w-4}{w+5}=-\frac{9}{2} t+\ln \frac{2}{5} \\ & \Rightarrow \frac{w-4}{w+5}=e^{-\frac{9}{2} t+\ln \frac{2}{5}}=\frac{2}{5} e^{-\frac{9}{2} t}=0.4 e^{-4.5 t} * \end{aligned}$ | M1 <br> M1 <br> A1ft <br> M1 <br> M1 <br> E1 <br> [6] | separating variables substituting their partial fractions <br> integrating correctly (condone absence of $c$ ) <br> correctly evaluating $c$ (at any stage) <br> combining lns (at any stage) <br> www |
| (v) As $t \rightarrow \infty \quad \mathrm{e}^{-45 t} \rightarrow 0$ $\Rightarrow \quad w-4 \rightarrow 0$ <br> So long term speed is $4 \mathrm{~m} \mathrm{~s}^{-1}$. | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & {[2]} \end{aligned}$ |  |

